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Synchrony-breaking Bifurcation Analysis using Lattice of Balanced Colourings

In general coupled cell networks, cells and arrows can represent distinct individuals and different interactions, such as genes interacting with each other either by activation or inhibition. When we can assume almost identical individuals interact with each other in a similar way, regular networks give a general formalism, in which all the nodes (cells) and arrows (interactions) are of the same type, and every cell has the same number of input arrows (called the valency of the network). Synchrony-breaking is where a fully synchronized network loses coherence, and breaks up into multiple clusters of self-synchronized subnetworks, termed patterns of synchrony. In particular, robust patterns of synchrony (or balanced colourings), where any initial state with this synchrony continues to have the synchrony for all time, are a consequence of network topology itself rather than specific dynamics. The topology (interactions) of a regular network is expressed by an adjacency matrix with integer entries, which determines all balanced colourings of the network. It has been proven by Stewart (2007) that all balanced colourings have a partially ordered structure and form a complete lattice for a general coupled cell network.

In this talk, we show that lattices of balanced colourings facilitate the classification of synchrony-breaking bifurcations of regular networks. Where the adjacency matrices have simple eigenvalues, explicit forms of the lattice can be constructed using the eigenvectors and eigenvalues to define key building blocks (lattice generators) and associated lattice indices. Each lattice point corresponds to partial synchrony of the network, and using these lattice properties we show the existence of codimension-one synchrony-breaking steady-state bifurcating branches, and furthermore, the number of partially synchronized clusters for each branch.

As an example, a large number of regular networks are classified into a small number of lattice structures, in which networks share an equivalent clustering type. Indeed, some of these networks even share the same generic bifurcation behavior. This classification leads us to explore how regular networks that share the same generic synchrony-breaking bifurcation behavior are topologically related. In some cases, two networks share a common core subnetwork, and this subnetwork drives the remaining network cells. However, other pairs of networks cannot be decomposed in this way, yet still share the same generic synchrony-breaking bifurcation.

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