I will discuss several theorems that link synchronisation to recurrences and recurrences to networks. I will first show that recurrences determine a system’s dynamics up to a homeomorphism and in some cases up to an isometry (based on a theorem on distance-one-preserving maps by Beckman). This can be used to study synchronisation between different systems and to develop a statistical hypothesis test for synchronisation. I will show how this can be used to infer the coupling direction for small groups of oscillators.

I will then discuss an equivalence of recurrence and adjacency matrices. This will lead me to the question of how to reconstruct graphs (unit disk graph reconstruction) and especially how to estimate the minimal dimension for such a reconstruction; to achieve this I will study the spectra of “forbidden sub-graphs” for different dimensions and link them to the sphericity of the graph. This in turn can be used to analyse time series for synchronisation.