Hybrid models of balance

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Outline

1. **Introduction**
   - Motivation
   - Switched models of human balance
2. **A model with dead-zone and PD control**
   - Model
   - System dynamics
3. **Conclusions**

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Extracts from http://www.enotalone.com/article/10145.html

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- There are many types of balance disorders. One of the most common is benign paroxysmal positional vertigo, or BPPV.
- BPPV is more likely to occur in adults ages 60 and older, but can also occur in younger people.

Who do we model?

Our modelling holds for either gender...

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- A human subject is modelled as an inverted pendulum with the center of mass located at some distance h from the ground
- There is a threshold value above which an active control is applied yielding a switched model

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  the postural sway feedback $\tilde{f}$ operates by allowing the system to drift for some small displacement with stabilizing negative feedback (closed loop control) switched on when the displacement exceeds certain threshold value

Reduction to a first order model

- For healthy subjects with eyes open $\gamma\dot{\phi} \gg mR^2\ddot{\phi}$, and after an appropriate rescaling we obtain

$$\dot{x} = \begin{cases} x + \sqrt{2D}\xi(t) + C & x(t-\tau) < -1, \\ x + \sqrt{2D}\xi(t) & -1 \leqslant x(t-\tau) < 1, \\ x + \sqrt{2D}\xi(t) - C & x(t-\tau) > 1, \end{cases}$$

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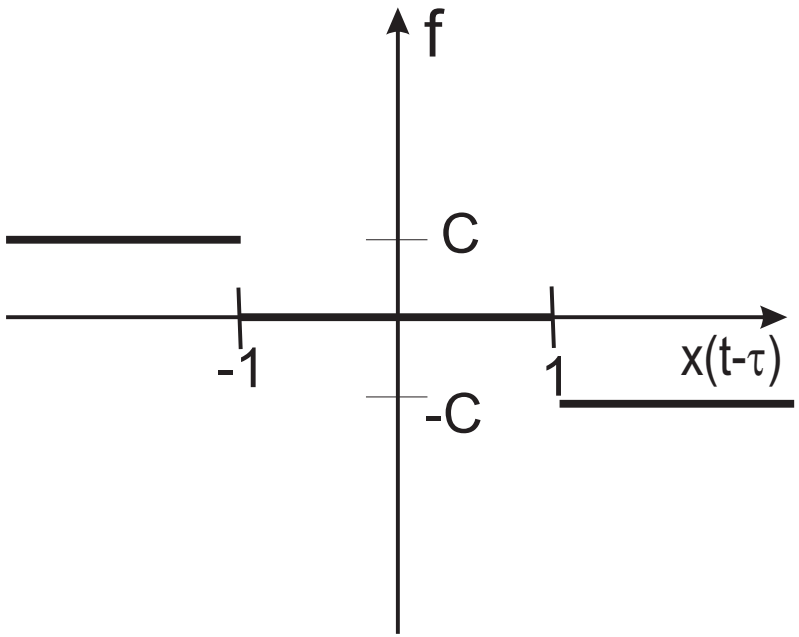
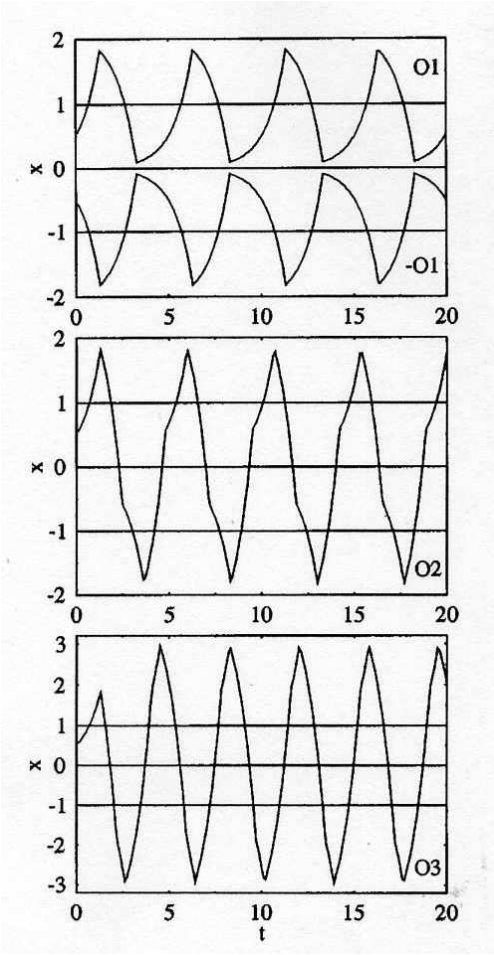
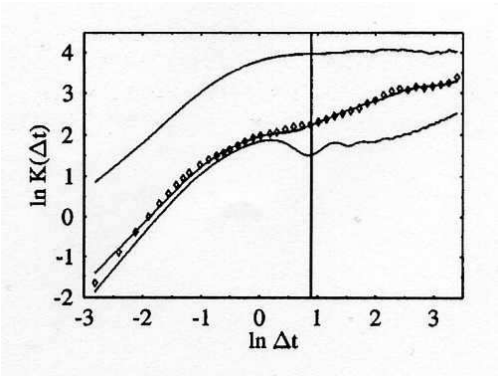


Dynamics

Stable periodic oscillations



Correlation function



We conjecture that there are two essential features to produce a correlation function with multiple scaling regions: (1) two or more coexisting limit cycle attractors in which transitions between attractors occur only at certain phases of the cycle; (2) noise of sufficient intensity to cause transition between the attractors at not too high a rate.

Alternative models, Asai et al. 2009

- The human upright posture is modelled by the motion of an inverted pendulum

$$I\ddot{\theta} = mgh\theta - (K\theta + B\dot{\theta} + f_P(\theta_\Delta) + f_D(\dot{\theta}_\Delta)) + \sigma\xi$$

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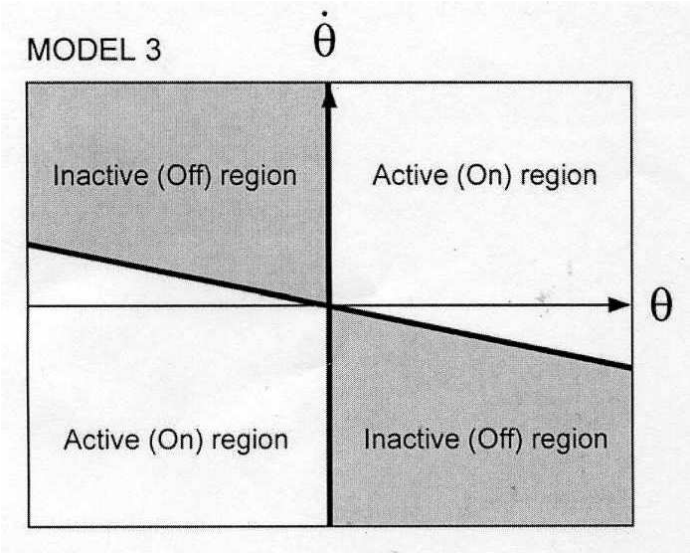
Switched control



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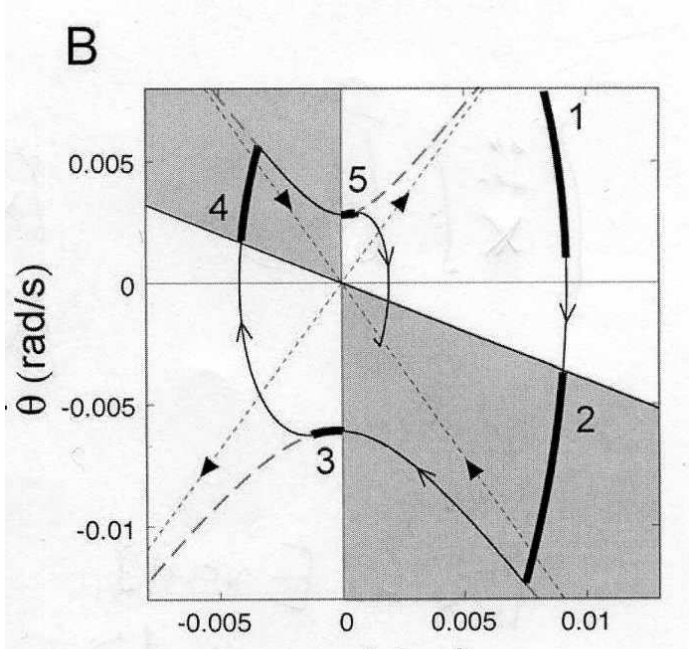

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$$J\ddot{\phi} = mgh\sin(\phi) \qquad \text{for} \qquad |\phi| \leqslant \phi_0,$$

$$J\ddot{\phi} = mgh\sin(\phi) + K_p\phi(t-\Delta_1) + K_D\dot{\phi}(t-\Delta_2) \qquad \text{for} \qquad |\phi| > \phi_0$$

Nondimensional model

Let $\bar{t} = \sqrt{\frac{g}{h}}t$, $\phi(t) = \theta(\bar{t})$, $x(\bar{t}) = \frac{d\theta(\bar{t})}{d\bar{t}}$, $A = \frac{mh^2}{J}$, $B = \frac{hK_p}{gJ}$, $C = \frac{\sqrt{h}K_D}{\sqrt{g}J}$, $\tau_1 = \frac{\sqrt{g}}{\sqrt{h}}\Delta_1$, and $\tau_2 = \frac{\sqrt{g}}{\sqrt{h}}\Delta_2$.

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$$\begin{pmatrix} \dot{\theta}(\bar{t}) \\ \dot{x}(\bar{t}) \end{pmatrix} = L \begin{pmatrix} \theta(\bar{t}) \\ x(\bar{t}) \end{pmatrix}, \qquad \text{for} \qquad |\theta| \leqslant \theta_0, \tag{1}$$

and for $|\theta| > \theta_0$,

$$\begin{pmatrix} \dot{\theta}(\bar{t}) \\ \dot{x}(\bar{t}) \end{pmatrix} = L \begin{pmatrix} \theta(\bar{t}) \\ x(\bar{t}) \end{pmatrix} + \begin{pmatrix} 0 \\ B\theta(\bar{t} - \tau_1) + Cx(\bar{t} - \tau_2) \end{pmatrix} \tag{2}$$

where

$$L = \begin{pmatrix} 0 & 1 \\ A & 0 \end{pmatrix}. \tag{3}$$

No delay

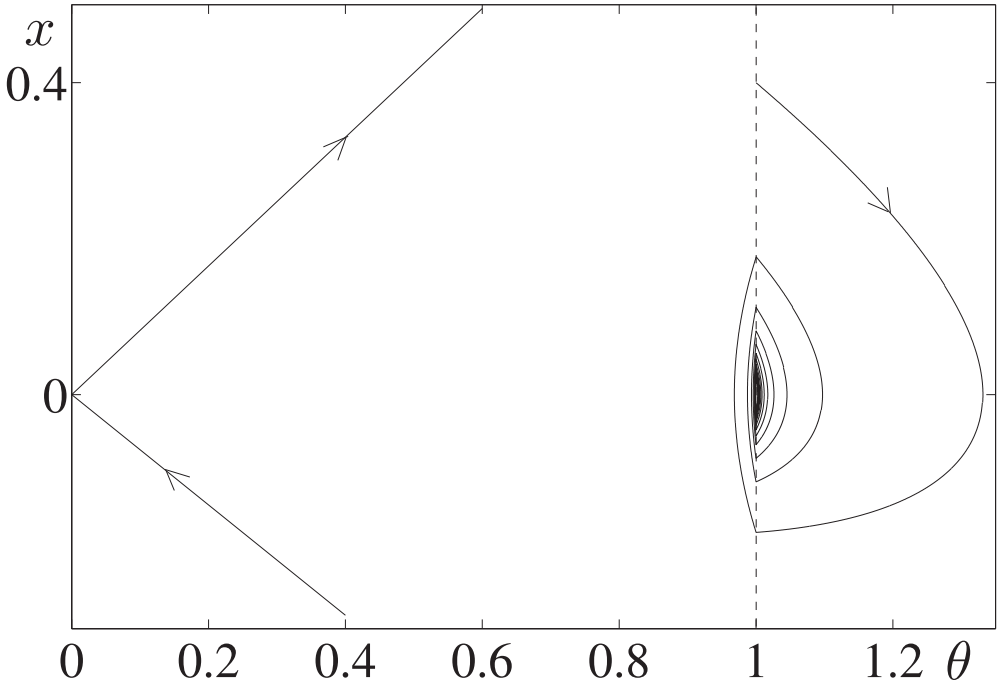

Figure: Illustrative trajectory of switched system (1) and (2) for $A = 0.5$, $B = -0.6$, $C = -0.5$, $\theta_0 = 1$, and the initial conditions $(\theta_0, x_0) = (1, 0.4)$ converging to a pseudo-equilibrium $(\theta_0, 0)$. The eigenvectors are depicted as solid lines.

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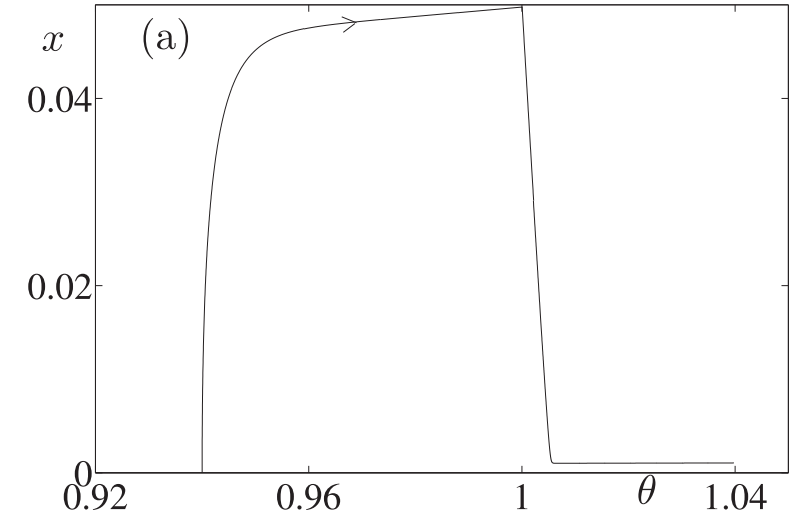
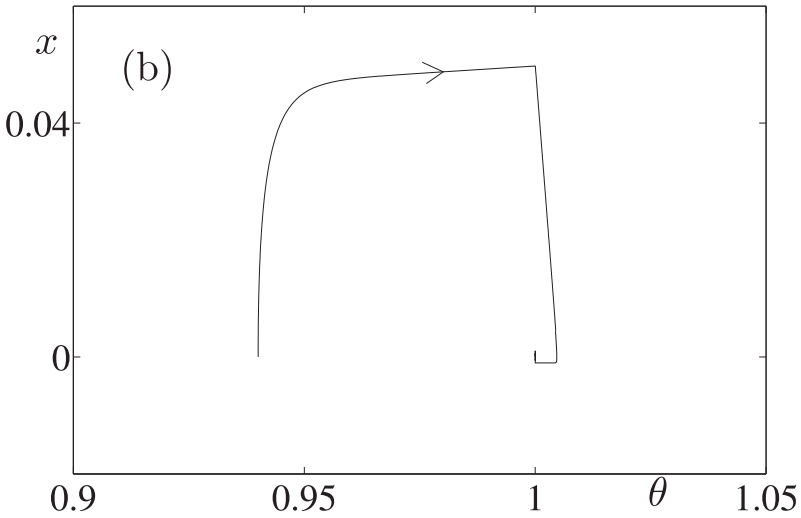
- However the angle θ at which the body is stabilized is offset from the vertical angle $\theta = 0$
- Comparison with the Eurich and Milton model reveals that, by modifying our switched model by adding $C\dot{\theta}(t)$ within the dead zone, depending on the sign of $A + B$ a pseudo-equilibrium is either stable or unstable

Adding the damping term





Small time delays with position or velocity feedback only

- If the derivative control is not active ($C = 0$: the damping term not present) or if the proportional control is not active ($B = 0$: no additional stiffness), and the time delay is small enough, the switched system is not controllable, and all the trajectories diverge to $\pm\infty$;

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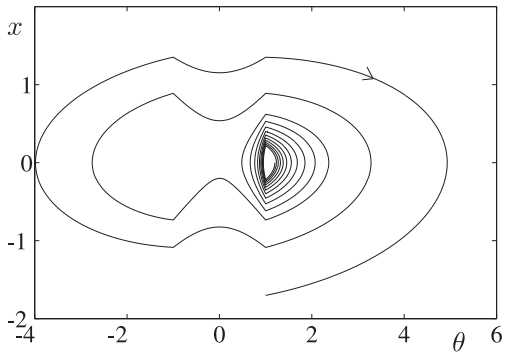

Figure: Representative trajectory of the switched system for $A = 0.5$, $B = -0.6$ and $C = 0$; time delay $\tau = 0.1$.

Multistability, PD control on

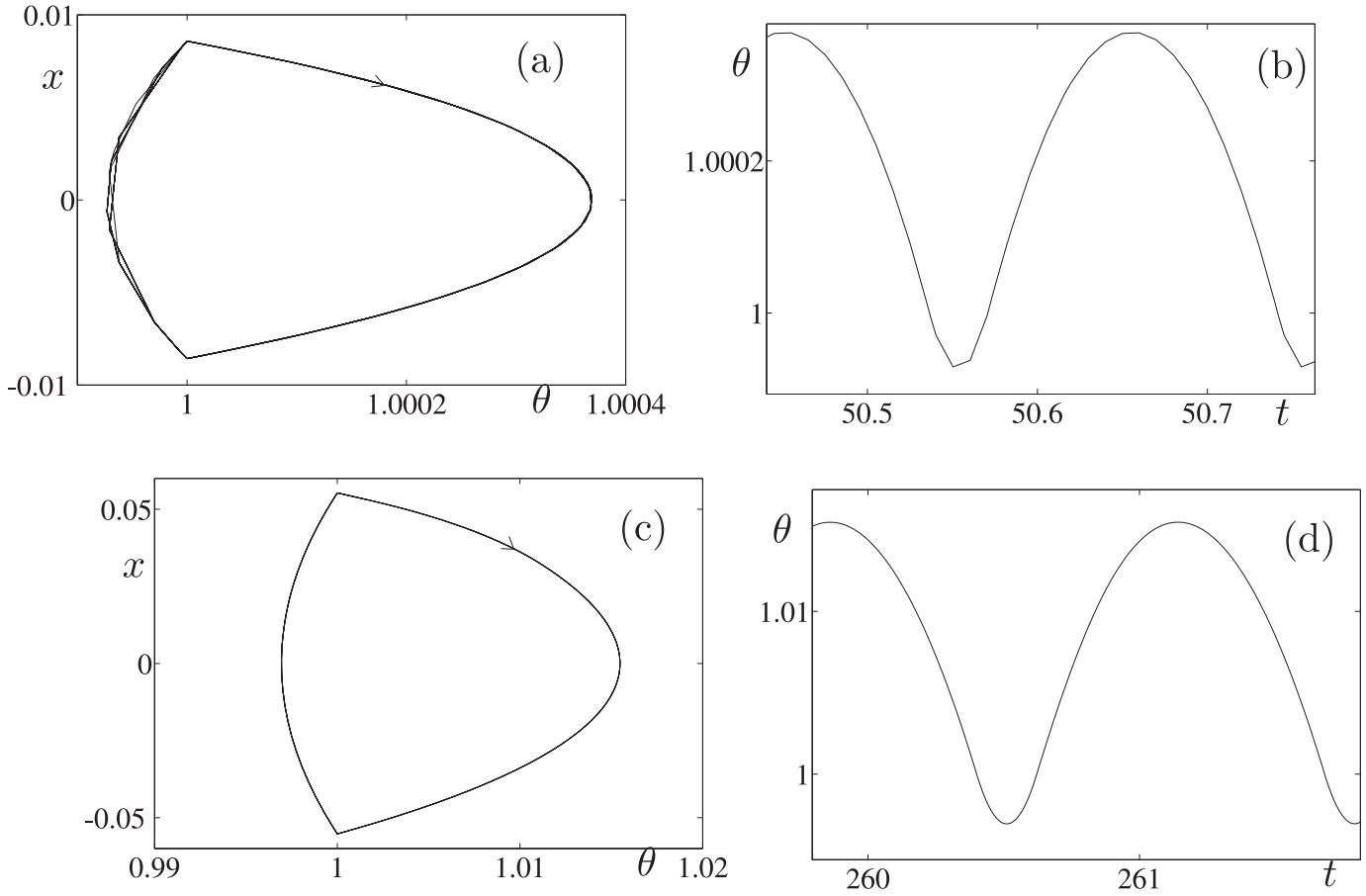

Figure: Co-existing stable limit cycles and corresponding time series for $A = 0.5$, $B = -0.6$ and $C = -0.5$. Time delay $\tau = 0.25$.

Bifurcation diagram

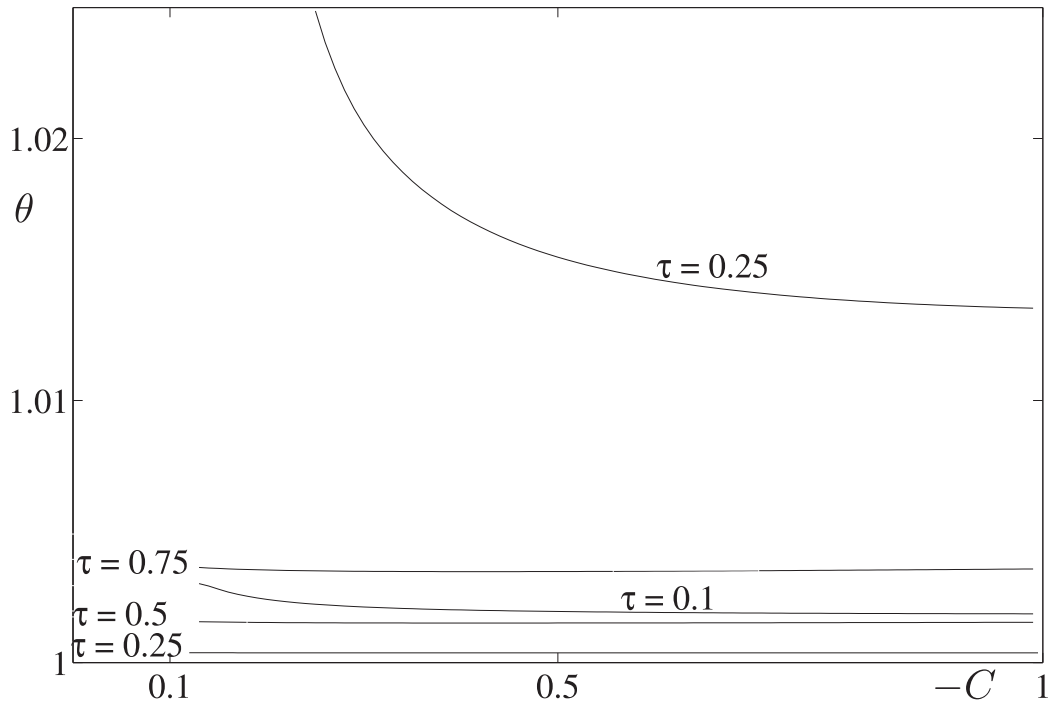

Figure: One-parameter bifurcation diagram capturing two families of periodic attractors. For one family the values of τ are given to the left of the curves, and for the second family the values of τ are given above the curves).

Homoclinic bifucations

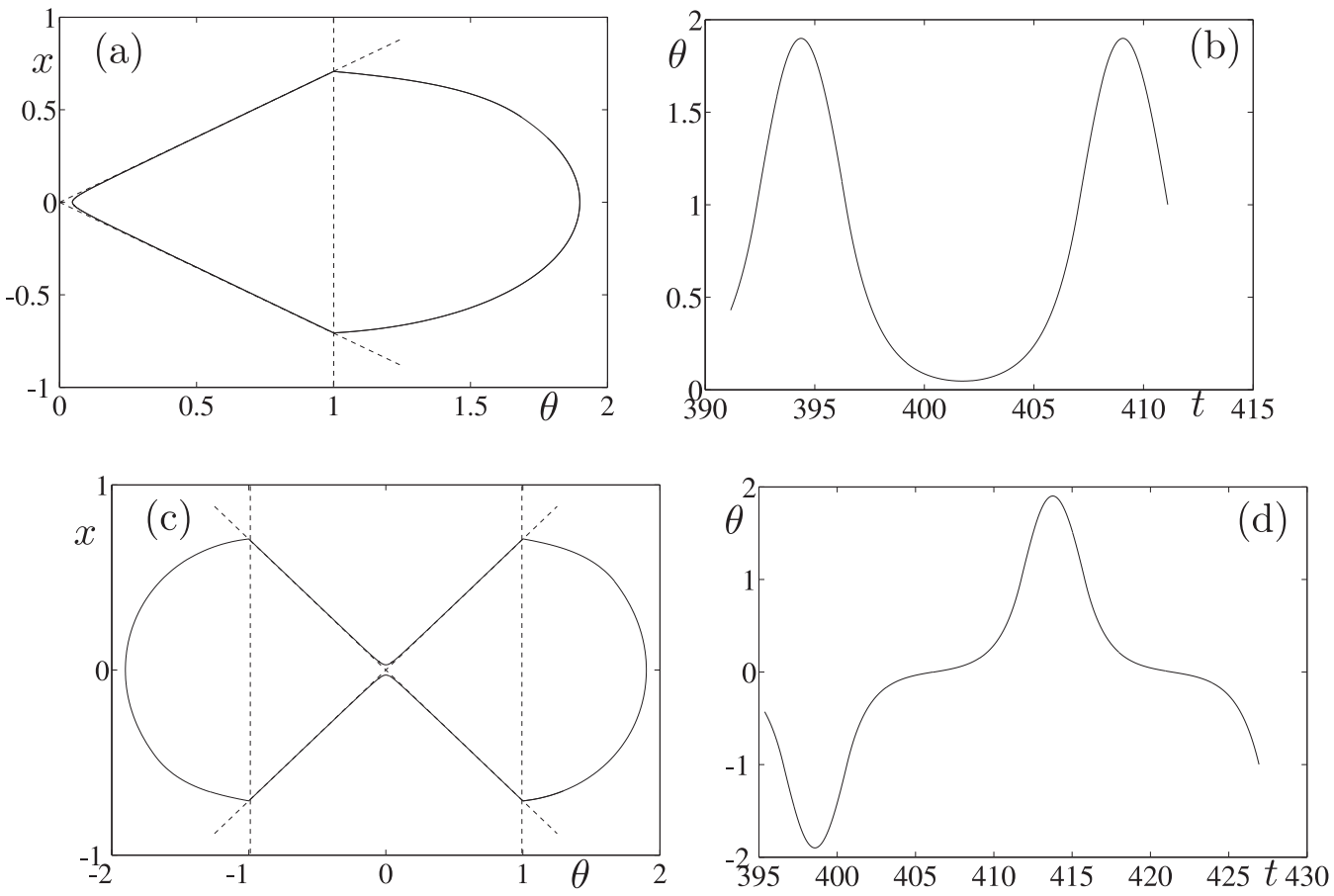

Figure: Homoclinic bifurcation for $A = 0.5$, $B = -0.6$, $C = -1$; (a) 'before' the bifurcation for $\tau = 1.1135$, and (c) after the bifurcation for $\tau = 1.114$.

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- Our parameter study leads to the detection of ‘true’ multistability
- Experimental verification of the stable oscillatory states found in our model system

Relation to physical parameters

if we take physiologically feasible values, and set $m = 60kg$, $h = 1m$, $g = 9.81m/s^2$, $J = 60kgm^2$, time delay $\Delta_1 = \Delta_2 = 0.1s$, the width of the dead zone $|\theta| = 0.02rad$, and the control coefficients $K_p = 720Nm/rad$, $K_D = 60Nms/rad$ our nondimensional quantities are equal to $A = 0.0001$, $B = 0.00012$, $C = 0.0031$ and the time delay $\tau = 31$. Our numerical simulations show that for these parameter values the system exhibits stable pseudo-equilibria.